

D.E.'s

## Lesson 21 - First-Order Linear Differential Equations

### I. Definitions and Reminders

II Finding general solutions to 1st-order linear D.E.'s  
in standard form.

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### Announcement

Take-home quiz # Due @ start of class on  
Mon 10/23

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### Definitions/Reminders

① A D.E. is a 1st-order linear D.E. if it can be written in the form

$Ax + By + C \Rightarrow$   
line

$$A(x) \frac{dy}{dx} + B(x)y + C(x) = 0$$

keys: ①  $y$  only appears in  $\frac{dy}{dx}$  &  $y$

The standard form for a 1st-order linear D.E. is

$$\frac{dy}{dx} + P(x)y = Q(x)$$

② Everything else only involves  $x$

TODAY - Solve 1st-order linear D.E.'s that are already in standard form.

Ex Separable? 1st-order linear? Both? Neither?

(A)  $\frac{dy}{dx} + x^2y = 2x^2$  linear  
also separable

$$\frac{dy}{dx} = 2x^2 - x^2y$$

$$\frac{dy}{dx} = x^2(2-y)$$

(B)  $\cos(x) \frac{dy}{dx} + 2xy = x^3 + 2$  linear only

$\cos(x) \frac{dy}{dx} + 2xy - (x^3 + 2) = 0$

$\frac{dy}{dx} = \underbrace{\frac{x^3 + 2 - 2xy}{\cos(x)}}_{\text{Not separable}}$   
can't write RHS as  $f(x)g(y)$

(C)  $\frac{dy}{dx} = xy^2$  separable  
Not 1st-order linear  
because of square ( $y^2$ )  
on  $y$ .

### Reminders

①  $\frac{d}{dx} [f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$

②  $\int \frac{d}{dx} [f(x)] dx = f(x) + C$

③  $\frac{d}{dx} \left[ \int f(x) dx \right] = f(x)$

④  $\frac{d}{dx} \left[ e^{\int f(x) dx} \right] = e^{\int f(x) dx} \cdot \frac{d}{dx} \left[ \int f(x) dx \right] = e^{\int f(x) dx} f(x)$  chain rule

$$\text{Std form: } \frac{dy}{dx} + P(x)y = Q(x)$$

II. Finding sol'n's of 1st-order linear D.E.'s  
in standard form:

[Ex]

$$\frac{dy}{dx} + 3y = 5$$

1st order linear

$$\text{std form: } P(x) = 3$$

$$Q(x) = 5$$

① We are going to make LHS look like it came from product rule.

Rabbit: Multiply both sides by  $e^{3x}$

$$\underbrace{e^{3x} \frac{dy}{dx} + 3e^{3x} y}_{\text{Do you see product rule?}} = 5e^{3x}$$

integrating factor.

$$\frac{d}{dx}[e^{3x} y]$$

$$\frac{d}{dx}[e^{3x} y] = 5e^{3x}$$

$$\int \frac{d}{dx}[e^{3x} y] dx = \int 5e^{3x} dx$$



$$e^{3x} \cancel{y}^u = \frac{5e^{3x}}{3} + C$$

integrating factor

$$y = \frac{5/3 e^{3x} + C}{e^{3x}}$$

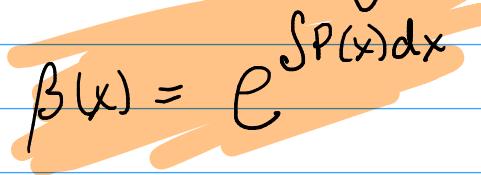
integrating factor

$$y = \frac{5}{3} + \frac{C}{e^{3x}} \quad \text{OR} \quad y = \frac{5}{3} + Ce^{-3x}$$

Procedure for solving 1st-order linear D.E.'s in std form

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (*)$$

① Calculate the integrating factor



$$\beta(x) = e^{\int P(x) dx}$$

Don't add  
constant to  
 $\int P(x) dx$

Last ex:  $P(x) = 3$

$$\beta(x) = e^{\int 3 dx} = e^{3x}$$

② Multiply both sides of (\*) by  $\beta(x)$

$$e^{\int P(x) dx} \frac{dy}{dx} + e^{\int P(x) dx} P(x)y = e^{\int P(x) dx} Q(x)$$

1st
2nd'
1st'
2nd

③ Integrate both sides to find sol'n

$$e^{\int P(x) dx} y = \int e^{\int P(x) dx} Q(x) dx$$



$$\beta(x)y = \int \beta(x)Q(x)dx$$

④ Divide by  $\beta(x)$  to solve for  $y$ .

$+C$  when  
you do  
 $\int$  on RHS

Ex (Edwards, et.al §1.5 #4)

$$\frac{dy}{dx} - 2xy = e^{x^2}$$

1st-order linear

std form  $\textcircled{U}$

$$P(x) = -2x$$

$$Q(x) = e^{x^2}$$

$$\textcircled{1} \quad \beta(x) = e^{\int -2x dx} = e^{-x^2}$$

$$\textcircled{3} \quad ye^{-x^2} = \int \underbrace{e^{-x^2}}_{e^{-x^2+x^2}} e^{x^2} dx = e^0 = 1$$

$$ye^{-x^2} = \int 1 dx$$

$$ye^{-x^2} = x + C$$

$$\textcircled{4} \quad y = \frac{x + C}{e^{-x^2}} = \frac{x}{e^{-x^2}} + \frac{C}{e^{-x^2}}$$

$$y = xe^{x^2} + Ce^{x^2}$$

$$\frac{2y}{\sqrt{x}}$$

Ex  $\frac{dy}{dx} + \frac{2}{x}y = x^4$

$$\text{Ans: } y = \frac{x^5}{7} + \frac{C}{x^2}$$